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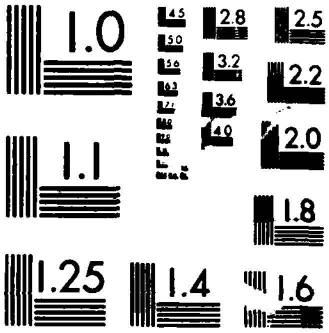

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This report considers packet radio performance in mobile fading channels. The first chapter considers the optimal packet size for NPCSMA scheme in mobile fading channels. Here, modified expression for the channel throughput of nonpersistent carrier sense multiple access protocol (NPCSMA) in slow Nakagami fading channel is given. The optimal pakcet size and the acknowledgement size are obtained such that a maximum number of users can share the channel while maintaining reliable values for both the probability of packet and acknowlegement error. Meanwhile the useful throughput is also insured to be at a maximum value. The second chapter

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SUMMARY

This report considers packet radio performance in mobile fading channels.

The first chapter considers the optimal packet size for NPCSMA scheme in mobile fading channels. Here, modified expression for the channel throughput of nonpersistent carrier sense multiple access protocol (NPCSMA) in slow Nakagami fading channel is given. The optimal packet size and the acknowledgement size are obtained such that a maximum number of users can share the channel while maintaining reliable values for both the probability of packet and acknowledgement error. Meanwhile, the useful throughput is also insured to be at a maximum value.

The second chapter considers packet radio performance over fast fading channels. Here, expressions for the throughput and packet delay of nonpersistent carrier sense multiple access (NPCSMA) protocol in fast fading environment are derived. Upper and lower bounds are obtained for a case where the fading phenomena is neither slow nor fast. An alternative technique is used to approximately predict the performance for the cases that exist between these bounds. Both Nakagami fading and noncoherent reception are assumed.

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CHAPTER I

OPTIMAL PACKET SIZE FOR NPCSMA SCHEME

IN MOBILE FADING CHANNELS

I. Introduction

Packet switching started with the development of the ARPANET [1]. The ALOHA protocol [2] was the first to employ radio links for packet communication. However it was established [3] that NPCSMA protocol is more adequate for ground -based communication and limited geographical region. However, the early analysis did not include the errors induced by Nakagami fading. Modified expression for the channel throughput for NPCSMA protocol is introduced. This last expression is found to depend on both the packet size and the acknowledgement size through the probability of packet and acknowledgement error. Optimal values of packet size and acknowledgement size are obtained by solving numerically two time-varying linear differential equations. The optimality criterion is chosen to be the maximum number of users that are simultaneously using the channel. We also guarantee the probability of packet as well as the acknowledgement error to be confined to some satisfactory values. Optimal packet size problem has been addressed before [4,5,6]. However none of these analyses considered the number of users to be the criterion of optimality.

II. Throughput Analysis for NPCSMA in Slow Nakagami Fading

The channel throughput (S) is defined to be the average number of packets successfully transmitted in T seconds, where T is the length of the packet. The total offered traffic rate on the channel is denoted by G. The probability of success on any packet transmission is P_S where

$$P_S = \frac{S}{G} = \frac{\exp(-aG)}{G(1+2a) + \exp(-aG)} \tag{1}$$

where a is the normalized round trip delay. If both the packet channel and the acknowledgement channel are fading, then the probability of packet success is given by

$$P_S = \frac{\exp(-aG)}{\exp(-aG) + G(1+2a)} (1 - \bar{P}_{pe})(1 - \bar{P}_{ae}) \tag{2}$$

where $\bar{P}$ is the probability that at least one bit is received in error in a block of N bits, pe stands for packet error and ae for acknowledgement success.

The pdf of Nakagami is given by [8]

$$f_R(r) = \frac{2}{\Gamma(m)} (\frac{m}{\Omega})^m r^{2m-1} \exp(\frac{-mr^2}{\Omega}) , r \geq 0 \tag{3}$$

where

$$\Omega = \overline{r^2}\ , \ m = \frac{\overline{r^2}}{\overline{(r^2 - \overline{r^2})^2}} \geq \frac{1}{2} \tag{4}$$

m is known as the fading figure. It indicates how severe the fading is.

For NCFSK type of modulation and slow Rayleigh fading, the average probability of block error $(\overline{P})$ is given by:

$$\overline{P} = 1 - \int_o^\infty (1 - \frac{1}{2}\ e^{-r^2/2N_o})^N\ \frac{2}{\Gamma(m)}\ (\frac{m}{\Omega})^m$$

$$\cdot\ r^{2m-1}\ \exp(\frac{-mr^2}{\Omega})\ dR \tag{5}$$

Substituting for $\gamma = \frac{r^2}{2N_o}$ in (5) where $2N_o$ is the spectral height of the additive white Gaussian noise, one gets:

$$\overline{P} = 1 - \int_o^\infty (1 - \frac{1}{2}\ e^{-\gamma})^N\ \frac{2}{\Gamma(m)}\ k^m\ \gamma^{m-1}$$

$$\cdot\ \exp(-K\gamma)\ d\gamma \tag{6}$$

where

$$K = \frac{m}{\overline{\gamma}} \tag{7}$$

$$\begin{aligned}\overline{\gamma} &= \text{Average SNR} \\ &= \frac{\Omega}{2N_o}\end{aligned} \tag{8}$$

Now after simple manipulations, the modified channel throughput for NPCSMA in Nakagami fading can be shown to be equal to :

$$S_m = \frac{G\ \exp(-aG)}{\exp(-aG) + (1 + 2a)G} \sum_{n=0}^{N_p} \binom{N_p}{n} (-\ \frac{1}{2})^n$$

$$\cdot\ (\frac{K_p}{K_p + \frac{n}{2}})^{m_p} \sum_{n=0}^{N_a} \binom{N_a}{n}\ (-\ \frac{1}{2})^n\ (\frac{K_a}{K_a + \frac{n}{2}})^{m_a} \tag{9}$$

It is obvious that S_m depends mainly on three quantities namely N, $\overline{\gamma}$, m, where the suffix can be either (a) for acknowledgement or (p) for packet. The dependence on m, γ is shown in figures (1) and (2), where identical packet and acknowledgement channels are assumed.

III. Optimal Packet and Acknowledgement Size:

The number of users (U) is proportional to a quantity S_u which we will call the useful throughput where:

$$S_u = \frac{N_{dp}\, N_{da}}{(N_{dp} + N_{ov})(N_{da} + N_{ov})}\, S_m \tag{10}$$

where N_{ov} is the overhead bits for either the acknowledgement or the packet

N_d is the number of data bits

$$N_a = N_{ov} + N_{da} \tag{11}$$

$$N_p = N_{ov} + N_{dp} \tag{12}$$

The above expression for S_u is logical since we have S_u approaching S_m as both $N_{da} \gg N_{ov}$ and $N_{dp} \gg N_{ov}$. On the other hand, if either N_{da} or N_{dp} approach zero S_u tends to zero too. Therefore the number of users is given by:

$$U = \frac{C}{N_{ov} + N_d}\, S_u \tag{13}$$

where

$$C = \frac{R_b}{\theta} \tag{14}$$

R_b is the bit rate of each user in bits/sec., and θ is the poisson rate at which all users independently transmit their packets in messages/sec.

The problem can be formulated as follows: what are the optimal packet size $\hat{N}_d$ and acknowledgement size $\hat{N}_a$ such that U is maximized while constraining the sum of probability of errors of both the packet and the acknowledgement to an apriori value $\overline{\alpha}$?

The above problem can be solved using the method of Lagrange multiplier. The problem of optimizing the function $\phi_1(\cdot)$ subject to some constraint is equivalent to optimizing the function $\phi_1(\cdot)$ where

$$\phi_1(X_1,\ldots\ldots,X_n) = \phi(X_1,\ldots\ldots,X_n) + \lambda f(X_1,\ldots\ldots,X_n) \tag{15}$$

The constant λ is known as the Lagrange multiplier and the necessary condition for an optimum value of ϕ to exist becomes;

$$\frac{\partial \phi_1}{\partial X_i} = \frac{\partial \phi}{\partial X_i} + \frac{\partial f}{\partial X_i} = 0 \quad ,i = 1,\ldots,n \tag{16}$$

Applying equation (16) to our problem, we get:

$$\begin{aligned} \phi_1(N_a,N_p) &= U(N_a,N_p) \\ &+ \lambda[\overline{\alpha} - \overline{P}_{pe}(N_p) - \overline{P}_{ae}(N_a)] \end{aligned} \tag{17}$$

Substituting from equation (13) into (17), we obtain:

$$\phi_1(N_p,N_a) = C_1\left(\frac{N_{dp}\, N_{da}}{N_p^2\, N_a}\right)(1 - \overline{P}_{ae})(1 - \overline{P}_{pe})$$

$$+ \lambda[\bar{\alpha} - \bar{P}_{pe}(N_p) - \bar{P}_{ae}(N_a)] \tag{18}$$

where

$$C_1 = \frac{C\ G \exp(-aG)}{\exp(-aG) + (1 + 2a)^G} \tag{19}$$

is a constant that does not depend on either N_p or N_a. Applying the above technique with $X_1 = N_p$, $X_2 = N_a$, we obtain after some rearrangements the following two equations:

$$0 = \dot{\bar{P}}_{ae}[-\lambda - \frac{N_{da}\,N_{dp}}{N_p^2\,N_a}\,C_1\,(1 - \bar{P}_{pe})]$$

$$+ [\frac{N_{dp}\,N_{ov}\,C_1}{N_p^2\,N_a^2}\,(1 - \bar{P}_{pe})] - [\frac{N_{ov}\,N_{dp}}{N_a^2\,N_p^2}\,(1 - \bar{P}_{pe})] \tag{20}$$

$$0 = \dot{\bar{P}}_{pe}[-\lambda - (1 - \bar{P}_{ae})\frac{N_{dp}\,N_{da}}{N_a\,N_p^2}\,C_1]$$

$$- \bar{P}_{pe}[\frac{N_{da}\,C_1}{N_a\,N_p^2}\,(1 - \bar{P}_{ae})\,(\frac{2N_{ov} - N_p}{N_p})]$$

$$+ [\frac{N_{da}\,C_1}{N_a\,N_p^2}\,(1 - \bar{P}_{ae})\,(\frac{2N_{ov} - N_p}{N_p})] \tag{21}$$

The above two equations are analogous to two linear differential equations with time-varying coefficients [9], which can be solved numerically for $\hat{N}_p$, $\hat{N}_a$. Figure (3) shows the number of users versus N_{dp}, N_{da} for $\bar{\alpha} = 0.0002$, $N_{ov} = 10$ bits, $\gamma_p = \gamma_a = 30$ dB, $m_p = m_a = 2$, $R_b = 64$ Kb/sec., $\theta = 1$ message/sec. The maximum number of users is 1083 for $\hat{N}_p = 20$ bits, $\hat{N}_a = 31$ bits.

In Figure (4) another example is shown where the average SNR is decreased to 27 dB while the fading figure is increased to 2.5. This resulted in an optimal value of $\hat{N}_p = 21$ bits while $\hat{N}_a = 51$ bits.

IV. Conclusion

The performance of the NPCSMA protocol in the presence of slow Nakagami fading is evaluated. The effect of changing both the fading figure and the average SNR on the throughput is obtained in closed form. The optimal packet and acknowledgement sizes are obtained to maximize the number of users utilizing the channel. There is always a tradeoff between the number of users and the size of the packet. It is shown that for a maximum number of users to be obtained a short packet has to be used, to maintain reliable values for the probability of packet and acknowledgement error. However, we can obtain a longer but suboptimal packet size by imposing this constraint on the problem statement. Another alternative would be to cut a long packet into segments of shorter packets and send them successively.

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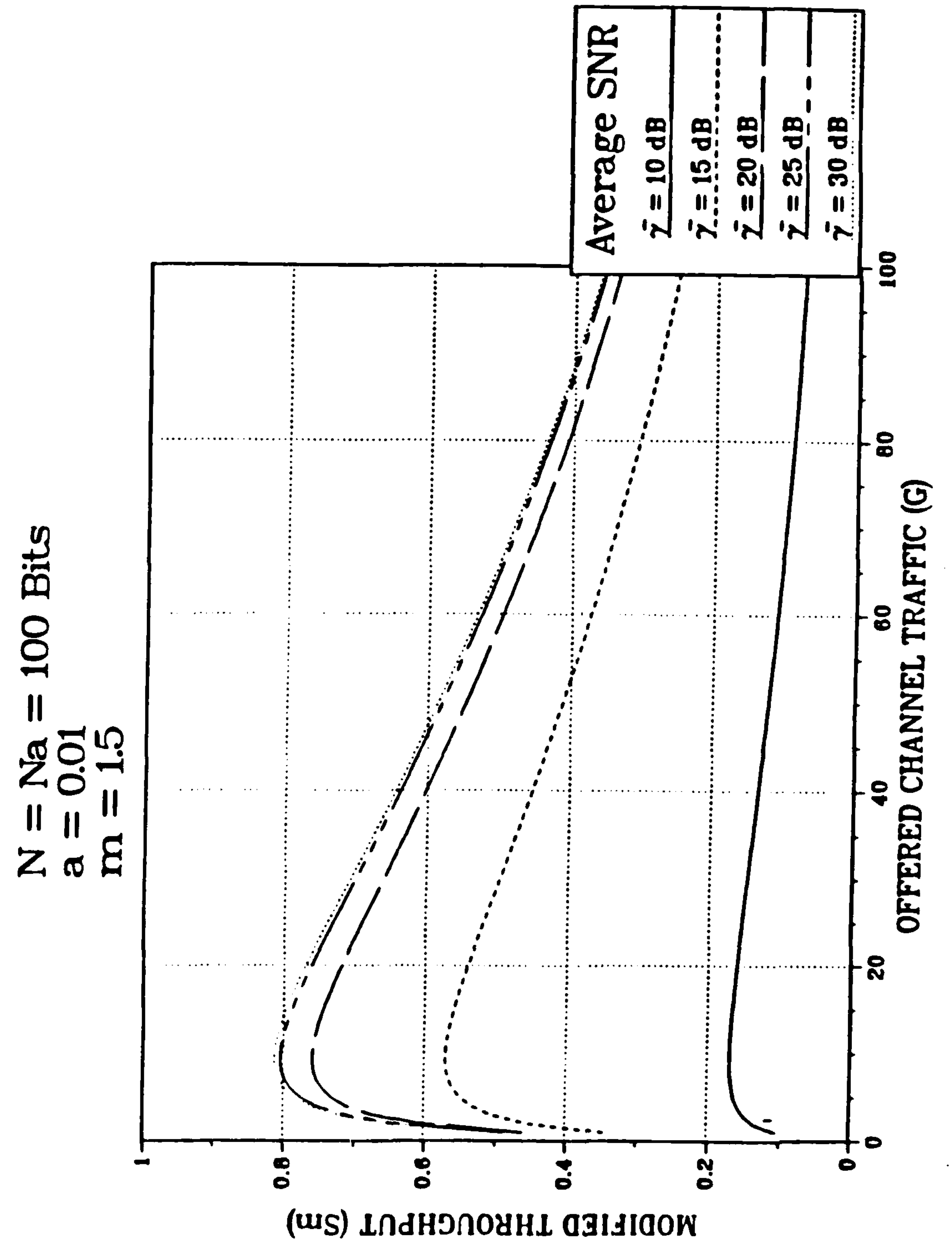

Figure 1 Throughput-Traffic characteristics for *NPCSMA* protocol in slow *Nakagami* fading for different *SNRs* .

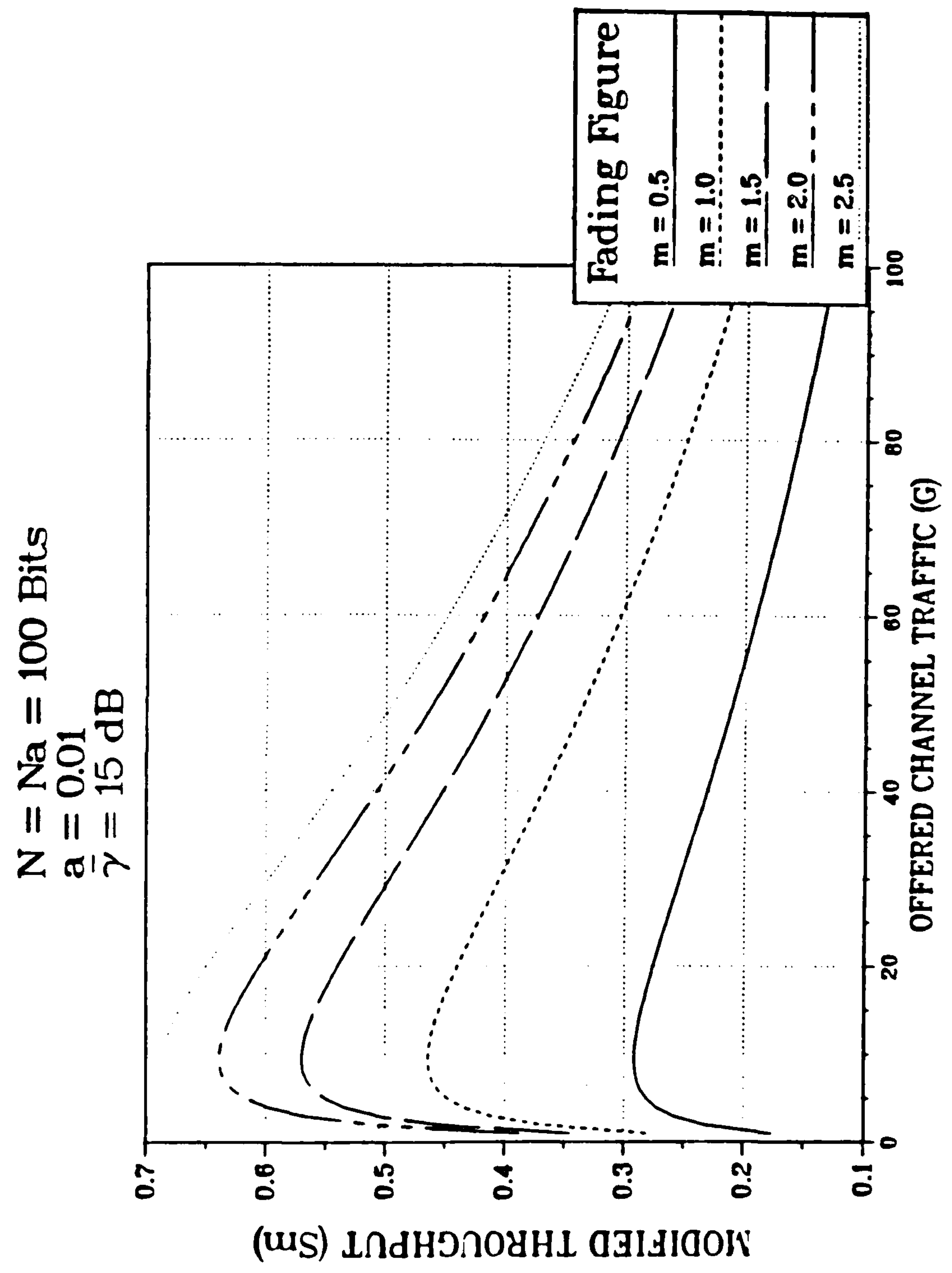

Figure 2 Throughput-Traffic characteristics for *NPCSMA* protocol in slow *Nakagami* fading for different fading figures.

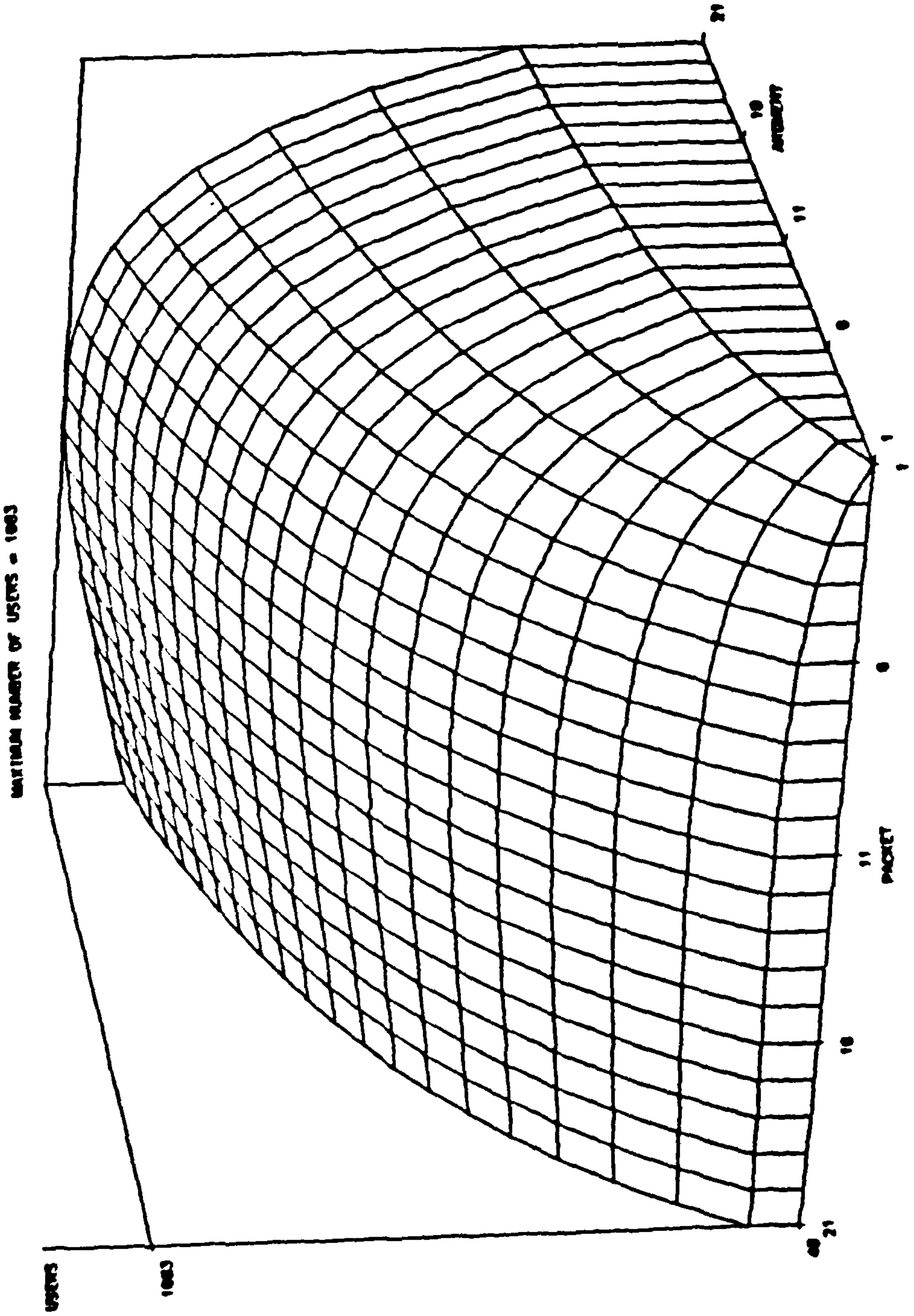

Figure 3 Number of users versus the acknowledgement size and the packet size: γ = 30dB, m = 2.0

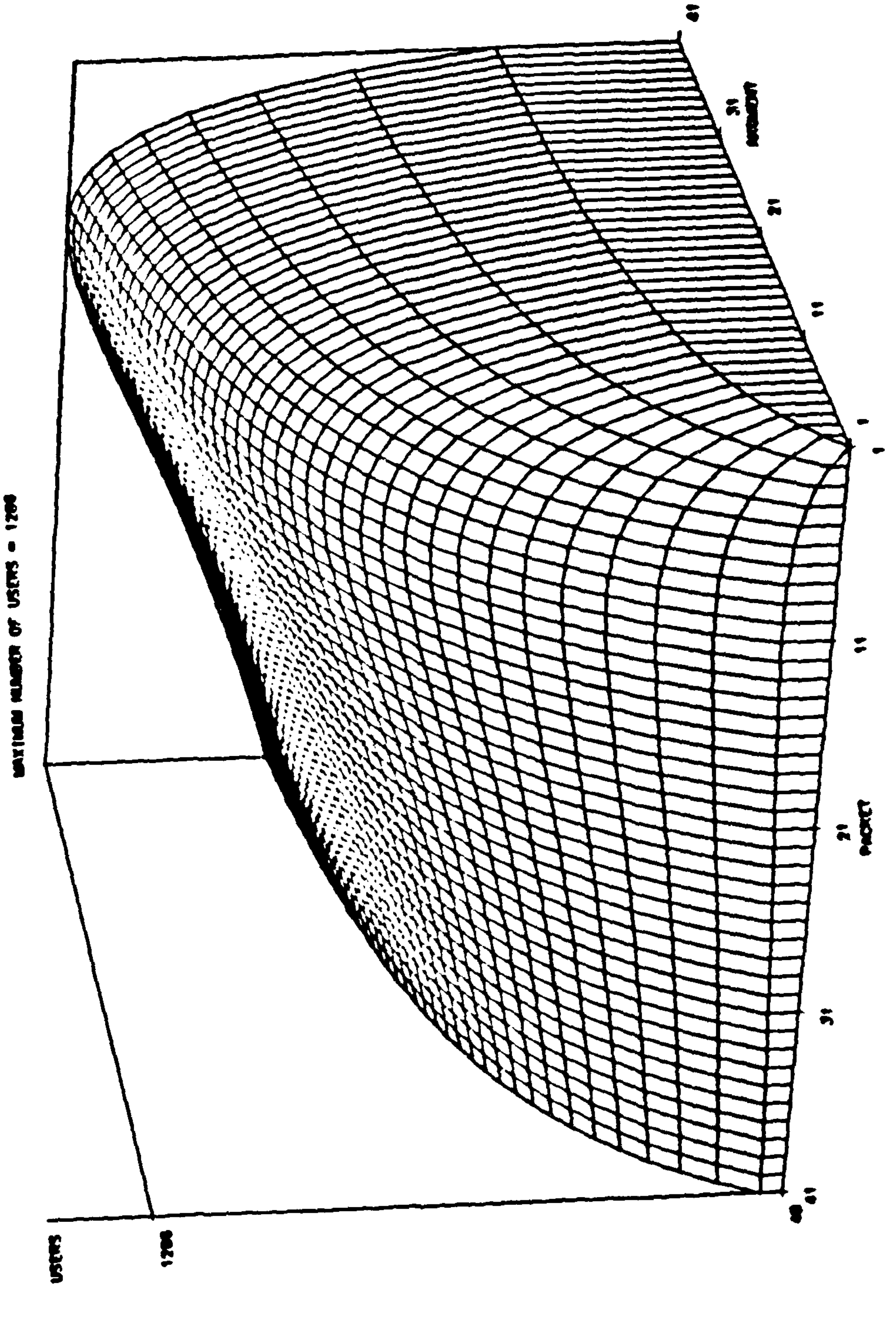

Figure 4 Number of users versus the acknowledgement size and the packet size: γ = 27 dB, m = 2.5

CHAPTER II

PACKET RADIO PERFORMANCE OVER FAST FADING CHANNELS

I. Introduction:

Both channel throughput and packet delay have been derived for the well-known random access schemes such as pure Aloha, slotted Aloha and NPCSMA [1-3]. The effect of errors due to fading was not considered. The performance of these protocols via slow Rayleigh fading channels have been considered [4,5]. In this paper, we investigate the situation where the signal intensity fluctuations are too quick with respect to the packet length that the slow fading assumption is no more valid. This allows us to assume independence between successive bit transmission within the same packet. The other situation that will be analyzed here occurs when the fading phenomena is neither too slow over the packet length nor too fast to assume independence of bit transmission. Within this situation, both Markov and Non simple Markov fading channels are considered. This analysis assumes a Nakagami [6] fading model which is more adequate in describing a wider class of fading modes. It has the advantage of including the Rayleigh as a special case. The modulation scheme was chosen to be NCFSK because it is more suitable for mobile communications.

II. NPCSMA Protocol with Nakagami Fading:

The channel throughput (S) is defined [7] to be the average number of packets transmitted in an interval equivalent to the packet duration ( ). For NPCSMA S is given by [3]:

$$S = \frac{G \exp(-aG)}{G(1+2a) + \exp(-aG)} \tag{1}$$

where (G) is the average channel traffic rate on the channel and (a) is the normalized one way propagation delay with respect to the packet duration. The corresponding expression for packet delay is also given by:

$$D = 1 + \alpha + 2a + (\frac{G}{S} - 1)(1 + \alpha + 2a + \delta) \tag{2}$$

where δ is the normalized average retransmission delay, and α is the normalized acknowledgement transmission time.

If the packet channel* is fading, the modified throughput will take the form:

$$S_m = (1 - \overline{P}_{pe})\, S \tag{3}$$

where $\overline{P}_{pe}$ is the average probability that a packet is received in error due to fading.

Similarly,

---

* We will assume throughout the analysis that the acknowledgement channel is independent from the packet channel and error-free.

$$Dm = 1 + \alpha + 2a + (\frac{G}{Sm} - 1)(1 + \alpha + 2a + \delta) \tag{4}$$

In the next section we will evaluate an expression for $\bar{P}_{pe}$ in fast Nakagami fading.

III. Probability of block error in fast Nakagami fading

The probability of block error is defined for slow fading by [8]

$$\bar{P}_{pe} = \int_{o}^{\infty} [1 - (1 - P(\gamma))^{N}]f(\gamma)\, d\gamma \tag{5}$$

where $f(\gamma)$ is the p.d.f. of the fading SNR.
$P(\gamma)$ is the probability of bit error for NCFSK, N is the number of bits in each packet.
However, for fast fading the SNR will vary from bit to bit and $\bar{P}_{pe}$ will be.

$$\bar{P}_{pe} = \int_{o}^{\infty} \cdots \int_{o}^{\infty} [1 - \prod_{i=1}^{N} (1 - P(\gamma_i))] \cdot f(\gamma_1, \cdots, \gamma_N)\, d\gamma_1 \cdots d\gamma_N \tag{6}$$

If the fading is fast enough, we can always assume independence between the SNR's over successive bits i.e.

$$\bar{P}_{pe} = 1 - \prod_{i=1}^{N} \int_{o}^{\infty} [1 - P(\gamma_i)]\, f(\gamma_i)\, d\gamma_i \tag{7}$$

For NCFSK and Nakagami fading:

$$P(\gamma_i) = \frac{1}{2} \exp(-\gamma_i/2) \tag{8}$$

,$f(\gamma_i)$ is given by [9]

$$f(\gamma_i) = \frac{K_i^{m}}{\Gamma(m_i)} \gamma_i^{m_i-1} \exp(-K_i\, \gamma_i) \tag{9}$$

where m_i is the fading figure of the i^{th} bit

$$, m_i = \frac{\Omega_i^2}{(R_i^2 - \Omega_i)^2} \geq \frac{1}{2} \tag{10}$$

$$, K_i = \frac{2N_o m_i}{\Omega_i} \tag{11}$$

, R_i is the i^{th} fading signal

, $2N_o$ is the spectral height of the

white Gaussian noise.

and Ω_i is the average power of the i^{th} signal.

Substituting from (8), (9) in (7), one can get,

or

$$\bar{P}_{pe} = 1 - \prod_{i=1}^{N} [1 - \frac{1}{2} (\frac{K_i}{K_i + \frac{1}{2}})^{m_i}] \tag{12}$$

The above expression can be evaluated for any given values of N, m_i, K sub i . Now, if the fading is neither fast enough to assume independence, nor slow enough for the SNR to remain constant over the packet length we have to modify the expression for the probability of block error. This will be done in the next section.

IV. Probability of block error in intermediate Nakagami fading:

The modified probability of block error in this case will become:

$$\bar{P}_{pem}(N) = 1 - \int_0^\infty \cdots \int_0^\infty f(\gamma_1, \cdots, \gamma_N) \prod_{i=1}^{N} (1 - \frac{1}{2} \exp - \gamma_i/2)\, d\gamma_1 \cdots d\gamma_N \tag{13}$$

At this point we will discuss two cases:

Case 1: The fading channel is Markov, viz.

$$\bar{P}_{pem}(N) = 1 - \int_0^\infty f(\gamma_1)(1 - \frac{1}{2} \exp - \gamma_1/2) \cdot \int_0^\infty f(\gamma_2/\gamma_1)(1 - \frac{1}{2} \exp - \frac{\gamma_2}{2}) \cdots \int_0^\infty f(\gamma_N \mid \gamma_{N-1})(1 - \frac{1}{2} \exp - \gamma_N/2)\, d\gamma_N \cdots d\gamma_1 \tag{14}$$

The above integral can be evaluated in closed form, though the computation will

be tedious for $N \geq 4$.* We will now discuss some special cases.

Case (a): Worst Correlation: $|\rho| = 1$.

To satisfy the above condition, the two signal-to-noise ratios must be linearly related [10]. In the special case where the means as well as the variances are the same for all SNR's, it can be shown [9] that

$$f(\gamma_l \mid \gamma_{l-1}) = \delta(\gamma_l - \gamma_{l-1}) \tag{15}$$

where $\delta(\cdot)$ is the delta function after Dirac.
Substituting from equation (16) into (15), one obtains:

$$\overline{P}_{pem} = 1 - \sum_{n=0}^{N} \binom{N}{n} \left(-\frac{1}{2}\right)^n \left(\frac{K}{K + \frac{1}{2}}\right)^m \tag{16}$$

which is the case of slow fading [9].

Case (b) Uncorrelated bits: ($|\rho| = 0$)

We will show that this case will reduce to the fast fading case.

Proof: The conditional density $f(\gamma_l \mid \gamma_{l-1})$ is given by [9]:

$$f(\gamma_l \mid \gamma_{l-1}) = \frac{K}{(1-\rho)\rho^{\frac{m-1}{2}}} \left(\frac{\gamma_l}{\gamma_{l-1}}\right)^{\frac{m-1}{2}}$$

$$\cdot \exp \frac{-K}{1-\rho} (\gamma_l + \rho\gamma_{l-1})\, I_{m-1} (K'' \sqrt{\gamma_l \gamma_{l-1}}) \tag{20}$$

where ρ is the power correlation coefficient between the two signals

$$, K'' = \frac{2K\sqrt{\rho}}{1-\rho}$$

, $I_x(\cdot)$ is the modified Bessel function of the first kind of order X. If we let $\rho = 0$ and expand the Bessel function one gets

$$f(\gamma_l \mid \gamma_{l-1}) = \frac{K^m \gamma_l^{m-1}}{\Gamma(m)} \exp - K\gamma_l = f(\gamma_l) \tag{18}$$

i.e. for any two jointly Nakagami random variables, uncorrelation implies independence.

Case (c) $|\rho|$ is small and low SNR's:

The conditional density in (17) can be approximated, using [11], by:

---

* See Appendix for more details.

$$f(\gamma_i \mid \gamma_{i-1}) \cong (\frac{K_i}{1-\rho})^{m_i} \frac{\gamma_i^{m_i-1}}{\Gamma(m_i)} \exp(-\frac{K_i}{1-\rho}\gamma_i) \tag{19}$$

This allows us to write $\overline{P}_{psm}(N)$ as:

$$\overline{P}_{psm}(N) = 1 - \prod_{i=1}^{N} \int_{o}^{\infty} (\frac{K_i}{1-\rho})^{m_i-1}$$

$$\cdot \exp(-\frac{K_i\gamma_i}{1-\rho})(1 - \frac{1}{2}\exp - \frac{\gamma_i}{2})\, d\gamma_i \tag{20}$$

or

$$\overline{P}_{psm}(N) = 1 - \prod_{i=1}^{N} [1 - \frac{1}{2}(\frac{K_i}{\frac{1}{2} + \frac{Ki}{1-\rho}})^{m_i}] \tag{21}$$

Case 2: Non Simple Markov Fading Channel (Threshold Analysis)

We will assume that a packet of N bits is successfully transmitted if the level of the fading SNR exceeds a prespecified threshold γ_{Th} at the first bit of the block and there occurs no downward crossings across this threshold. In probabilistic statements, one can write:

$$\begin{aligned}\overline{P}_{ps}(N) &\approx \text{Prob[No downward crossings across } \gamma_{Th}] \\ &\cdot \text{Prob[signal level} > \gamma_{Th}] \\ &\approx P_1 \cdot P_2\end{aligned} \tag{22}$$

The approximation takes place since we assume independence of the two events.

We start the analysis by stating some basic assumptions:

(1) The processes that are considered are stationary.

(2) The analysis does not apply to simple Markov processes.

(3) The noise level is normalized such that $2N_o = 1$.

(4) The occurrence of the crossings obey a Poisson distribution [10].

(5) The number of downward crossings is approximately one half that of the total number of crossings.

The last two assumptions allow us to write the second term in (22) as follows:

$$P_2 = \exp(-NT_b(\frac{N_{\gamma_{Th}}}{2})) \tag{23}$$

where T_b is the bit length in seconds, N_R is the total number of crossings across

the signal level R. It is known [6,12] that:

$$N_R = \int_0^{\infty} \dot{R}\, f(R,\dot{R},t)d\dot{R} \tag{24}$$

where $f(R,\dot{R},t)$ is the joint p.d.f. of the signal and its derivative. The previous equation was shown to reduce to:

$$N_R \cong \sqrt{-\frac{1}{4\pi}\, \ddot{\phi}\,(0)} \cdot f(R) \tag{25}$$

where f(R) is the Nakagami p.d.f. and $\phi(\tau)$ is the autocorrelation function of R.

$$, \ddot{\phi}\,(0) = -\,4\pi^2 \int_{-\infty}^{\infty} f^2 S(f)\, df \tag{26}$$

where S(f) is the power spectrum of R.
Substituting for $f(R)_{R_{Th}}$ in (31) we get:

$$N_R = \sqrt{\pi \int_{-\infty}^{\infty} f^2 S(f)\, df}$$

$$\cdot \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m R^{2m-1} \exp\left(-\frac{mR^2}{\Omega}\right) \tag{27}$$

where $\Omega = \bar{\gamma}$ = Average SNR according to the second assumption.
At the median level, we have:

$$N_{R_{med}} = \sqrt{\pi \int_{-\infty}^{\infty} f^2 S(f) df}$$

$$\cdot \frac{2}{\Gamma(m)} \left(\frac{m}{\Omega}\right)^m R_{med}^{2m-1} \exp\left(-\frac{mR_{med}^2}{\bar{\gamma}}\right) \tag{28}$$

Hence, in terms of $N_{R_{med}}$, (33) will take the form:

$$N_R = N_{R_{med}}\left(\frac{R}{R_{med}}\right)^{2m-1}$$

$$\exp\left[-\frac{mR_{med}^2}{\bar{\gamma}} \left\{\left(\frac{R}{R_{med}}\right)^2 - 1\right\}\right] \tag{29}$$

with

$$\frac{R}{R_{med}} = \sqrt{\frac{\gamma}{\gamma_{med}}} \tag{30}$$

Substituting for $N_{R_{med}}$ and arranging, one gets:

$$N_{\gamma} = \frac{2(m)^m\sqrt{\pi}}{\Gamma(m)} \left(\frac{\gamma}{\bar{\gamma}}\right)^{m-\frac{1}{2}} \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right) f_s \tag{31}$$

where f_N is given by [13]:

$$f_N = \frac{\int_{-\infty}^{\infty} f^2 S(f)df}{\sqrt{\int_{-\infty}^{\infty} S(f)df}} \tag{32}$$

at the threshold γ_{Th}, (29) becomes:

$$P_2 = \exp[-NT_b \frac{(m)^m\sqrt{\pi}}{\Gamma(m)} (\frac{\gamma_{Th}}{\gamma})^{m-\frac{1}{2}}$$

$$\exp(-\frac{m\,\gamma_{Th}}{\gamma})f_N] \tag{33}$$

One example which received considerable attention has the power spectrum:

$$S(\omega) = (\frac{2C}{C^2 + \omega^2})^2 \tag{34}$$

It represents the borderline between a simple Markov Process and the smoother processes like Gaussian. From (40), one can easily get for f_N

$$f_N = \frac{C}{2\pi} \tag{35}$$

The constant C is an indicator of how far the two random variables are correlated [10]. Hence, one can claim the following relationships

$CT_b \gg 1$ corresponds fast fading

. $CT_b \ll 1$ corresponds slow fading

. $T_b < \frac{1}{C} < NT_b$ corresponds intermediate fading

Returning to equation (22), the first term will be:

$$P_1 = \int_{\gamma_{Th}}^{\infty} f(\gamma)d\gamma = 1 - F(\gamma_{Th}) \tag{36}$$

where $F(\gamma)$ is the C.D.F. of the SNR, which can be shown to be

$$F(\gamma_{Th}) = P(m, K\gamma_{Th}) \tag{37}$$

where $P(b,x)$ is the incomplete Gamma function [11].
The choice of γ_{Th} must depend on the fading model as well as the modulation scheme. Moreover, the threshold must be chosen such that when $T_bC \ll 1$, the first term must approach the slow fading case that is obtained earlier. The above

conditions will be satisfied by equating the probability of packet error for the slow bound to $F(\gamma_{Th})$, i.e.,

$$F(\gamma_{Th}) = 1 - \sum_{n=0}^{N} \binom{N}{n} \left(-\frac{1}{2}\right)^n \left(\frac{K}{K + \frac{n}{2}}\right)^m \tag{38}$$

V. Throughput and packet delay expressions for NPCSMA:

For fast fading channel the modified throughput will be

$$S_m = \prod_{i=1}^{N} \left[1 - \frac{1}{2}\left(\frac{K_i}{K_i + \frac{1}{2}}\right)^{m_i}\right] \frac{G \exp -aG}{G(1+2a) + \exp(-aG)} \tag{39}$$

Similarly, the delay will take the form

$$D_m = 1 + \alpha + 2a + \left(\frac{G}{S_m} - 1\right)(1 + 2a + \alpha + \delta) \tag{40}$$

For intermediate fading, the results are

Case 1: Markov fading channel:

$$S_m = \int_0^\infty f(\gamma_1)\left(1 - \frac{1}{2}\exp -\frac{\gamma_1}{2}\right) \prod_{i=1}^{N} \cdot \left\{\int_0^\infty f(\gamma_i/\gamma_{i-1})\left(1 - \frac{1}{2}\exp -\frac{\gamma_i}{2}\right) d\gamma_i\right\} \cdot \frac{G \exp -aG}{G(1+2a) + \exp -aG} \tag{41}$$

$$D_m = 1 + \alpha + 2a + (1 + 2a + \alpha + \delta)\left(\frac{G}{S_m} - 1\right) \tag{42}$$

Case 2 Non Simple Markov fading channel

Substituting from equations (33), (37) in (22) one gets

$$S_m = [1 - P(m, k\gamma_{Th})] \cdot \exp\left[-\frac{Nm^m}{2\Gamma(m)\sqrt{\pi}}\left(\frac{\gamma_{Th}}{\gamma}\right)^{m-\frac{1}{2}}(CT_b) \cdot \exp\left(-\frac{m\gamma_{Th}}{\gamma}\right)\right] \cdot \frac{G \exp -aG}{G(1+2a) + \exp(-aG)} \tag{43}$$

$$D_m = 1 + \alpha + 2a + (1 + \alpha + 2a + \delta)\left(\frac{G}{S_m} - 1\right) \tag{44}$$

VI. Numerical Results

Figures [1-2] represent the performance index for NPCSMA with $N = 2$. It can be noticed that the best performance is achieved when the successive bits are completely correlated (slow fading). This implies that the degradation increases directly with the degree of rapidity of the fading signal. At larger values of N (100 bits), the slow and fast bounds are not enough to predict the performance at all ranges. However, Figures [3-6] show that for either higher in (fading figure) or average SNR, the bounds are close enough to simulate the performance accurately. Finally, Figures [7-8] display the effect of bit-to-bit correlation by changing the product ($C \cdot T_b$) between 0.01, 0.91, along with the two bounds (fast and slow). The threshold analysis has many advantages. It simplifies the complexity of computations for large N. It can also be as accurate as desired, depending on the choice of the increment by which CT_b changes. After all, it can be applied to any type of autocorrelation except for the simple Markov process where f_N does not exist.

VII. Conclusions

The performance of a NPCSMA protocol is analyzed in fast Nakagami fading. Expressions for the probability of packet error are obtained in fast and intermediate types of fading. In the latter case two cases are investigated namely, Markov and non simple Markov fading channels. Noncoherent frequency reception is assumed. This analysis can be applied to other access random schemes as well as other modulation modes.

Appendix A Probability of Block Error in Intermediate Nakagami-fading channel:

With the Markov assumption, we have for N = 1

$$\bar{P}_{pem}(1) = \frac{1}{2}\int_{o}^{\infty} f(\gamma_1)\exp(-\frac{\gamma_1}{2})d\gamma_1 \tag{A-1}$$

$$= \frac{1}{2}(\frac{K}{K+\frac{1}{2}})^m \tag{A-2}$$

If N = 2

$$\bar{P}_{pem}(2) = 1 - \int_{o}^{\infty} f(\gamma_1)[1 - \frac{1}{2}\exp - \frac{\gamma_1}{2}]$$

$$\cdot f(\gamma_2/\gamma_1)(1 - \frac{1}{2}\exp - \frac{\gamma_2}{2})\, d\gamma_2 d\gamma_1 \tag{A-3}$$

The second integral will be

$$I_2 = 1 - \frac{1}{2}\, L.T.[f(\gamma_2/\gamma_1)]_{S=\frac{1}{2}} \tag{A-4}$$

where L.T. is the Laplace transform operator. From [11], this can be evaluated to be

$$I_2 = 1 - \beta^m \exp[-\frac{K_\rho}{1-\rho}(1-\beta)\gamma_1] \tag{A-5}$$

where

$$\beta = \frac{K}{K+\frac{1-\rho}{2}} \leq 1 \tag{A-6}$$

Then

$$\bar{P}_{pem}(2) = 1 - \int_{o}^{\infty} f(\gamma_1)[1 - \frac{1}{2}\exp - \frac{\gamma_1}{2}]d\gamma 1$$

$$+ \beta^m \int_{o}^{\infty} f(\gamma_1)[1 - \frac{1}{2}\exp - \frac{\gamma_1}{2}]$$

$$\exp[-\frac{K\rho}{1-\rho}(1-\beta)\gamma_1]d\gamma_1 \tag{A-7}$$

After some effort $\bar{P}_{pem}(2)$ is evaluated to be:

$$\bar{P}_{pem}(2) = \frac{1}{2}(\frac{K}{K+\frac{1}{2}})^m$$

$$+ \frac{1}{2} \beta^m [(\frac{K}{K + \frac{K\rho}{1-\rho} (1-\beta)})^m$$

$$- \frac{1}{2} (\frac{K}{K + \frac{K\rho}{1-\rho} (1-\beta) + \frac{1}{2}})^m] \tag{A-8}$$

For $N = 3$

$$\overline{P}_{pem}(3) = 1 - \int_{o}^{\infty} f(\gamma_1)(1 - \frac{1}{2} \exp - \frac{\gamma_1}{2})$$

$$\cdot f(\gamma_2 \mid \gamma_1)(1 - \frac{1}{2} \exp - \frac{\gamma_2}{2})$$

$$\cdot \int_{o}^{\infty} f(\gamma_3 \mid \gamma_2)(1 - \frac{1}{2} \exp - \frac{\gamma_3}{2})\, d\gamma_3 d\gamma_2 d\gamma_1 \tag{A-9}$$

After some tedious calculations, we arrive at

$$\overline{P}_{pem}(3) = \frac{1}{2}(\beta\beta_1)^m [(\frac{K}{K + \frac{K\rho}{1-\rho}(1-\beta_1)})^m$$

$$- \frac{1}{2}(\frac{K}{K + \frac{K\rho}{1-\rho}(1-\beta_1) + \frac{1}{2}})^m]$$

$$+ \frac{1}{2}(\frac{K}{K + \frac{1}{2}})^m + \frac{1}{2} \beta^m [(\frac{K}{K + \frac{K\rho(1-\beta)}{1-\rho}})^m$$

$$- \frac{1}{2}(\frac{K}{K + \frac{K\rho}{1-\rho}(1-\beta_1) + \frac{1}{2}})^m]$$

$$- \frac{1}{4}(\beta\beta_2)^m [(\frac{K}{K + \frac{K\rho(1-\beta)}{1-\rho}})^m$$

$$- \frac{1}{2}(\frac{K}{K + \frac{K\rho}{1-\rho}(1-\beta_1) + \frac{1}{2}})^m] \tag{A-10}$$

where

$$\beta_1 = \frac{K}{K + K\rho(1-\beta)} = \frac{1}{1 + \rho(1-\beta)} \leq 1 \tag{A-11}$$

$$, \beta_2 = \frac{1}{1 + \rho(1-\beta) + \frac{1-\rho}{2K}} \leq 1 \tag{A-12}$$

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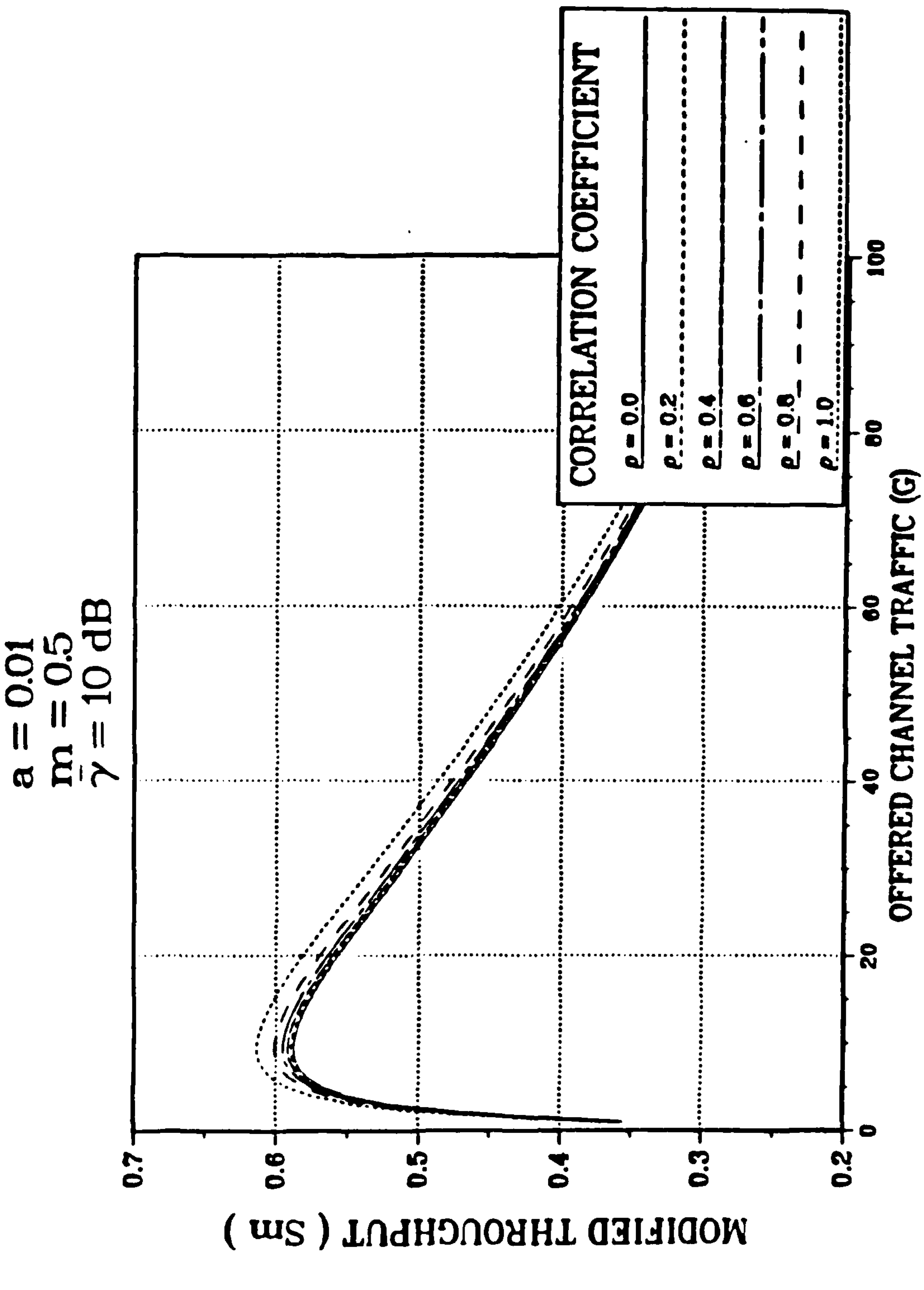

Figure 1 Effect of bit-to-bit correlation on the Throughput-Traffic characteristics for *NPCSMA* protocol (N=2).

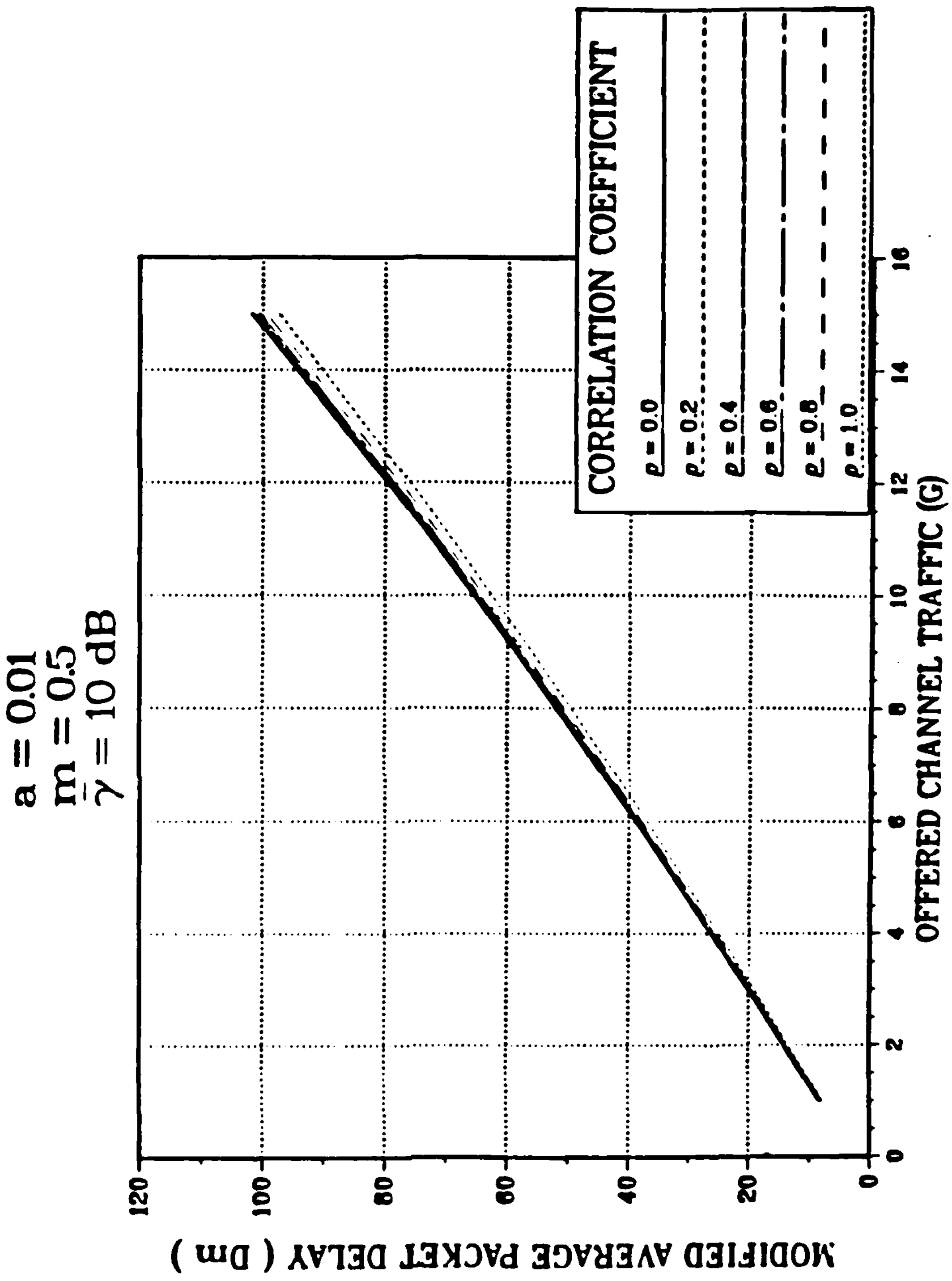

Figure 2 Effect of bit-to-bit correlation on the Delay-Traffic characteristics for *NPCSMA* protocol (N=2).

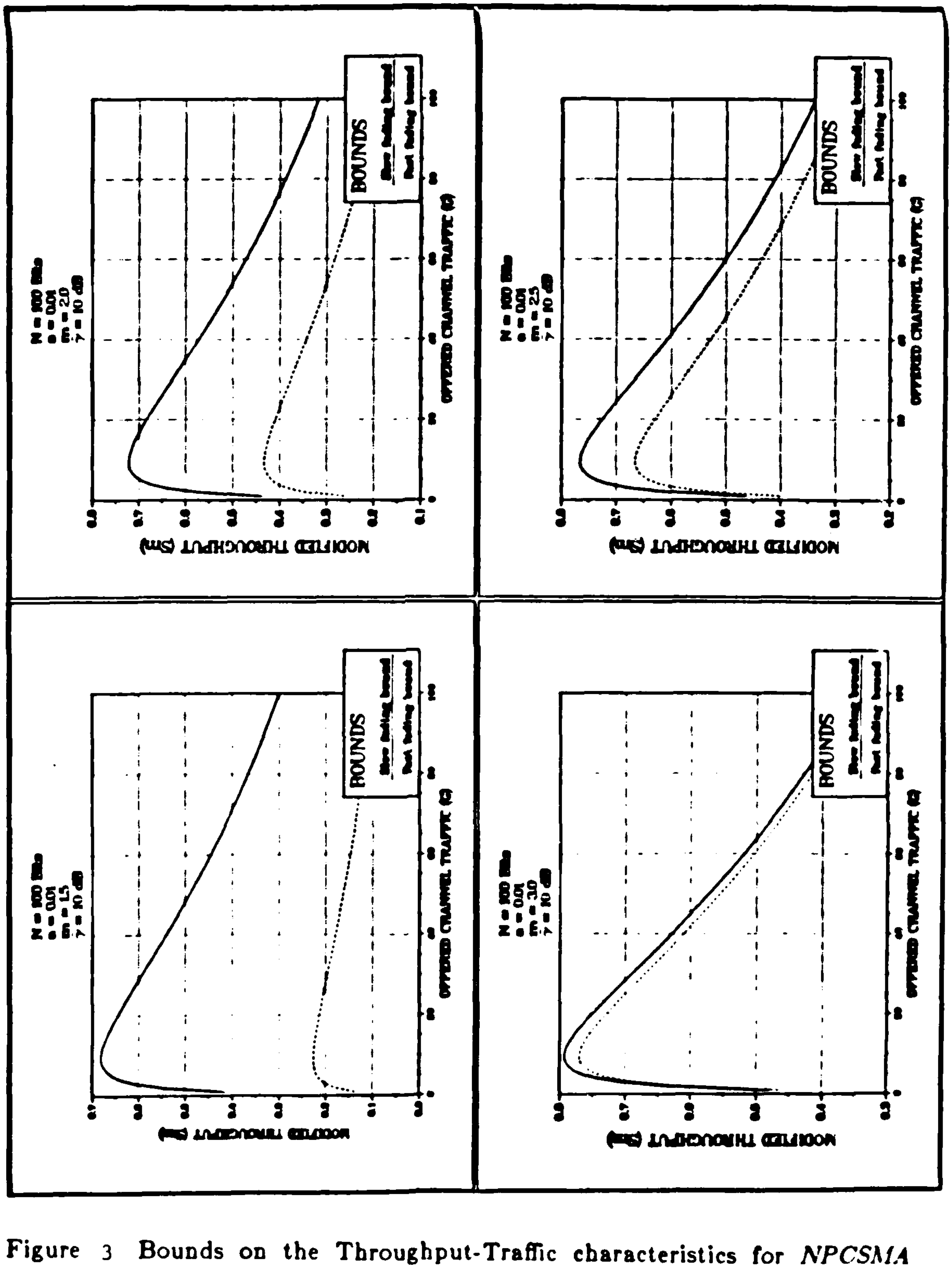

Figure 3 Bounds on the Throughput-Traffic characteristics for *NPCSMA* protocol (N=100) and fast *Nakagami* fading for different fading figures.

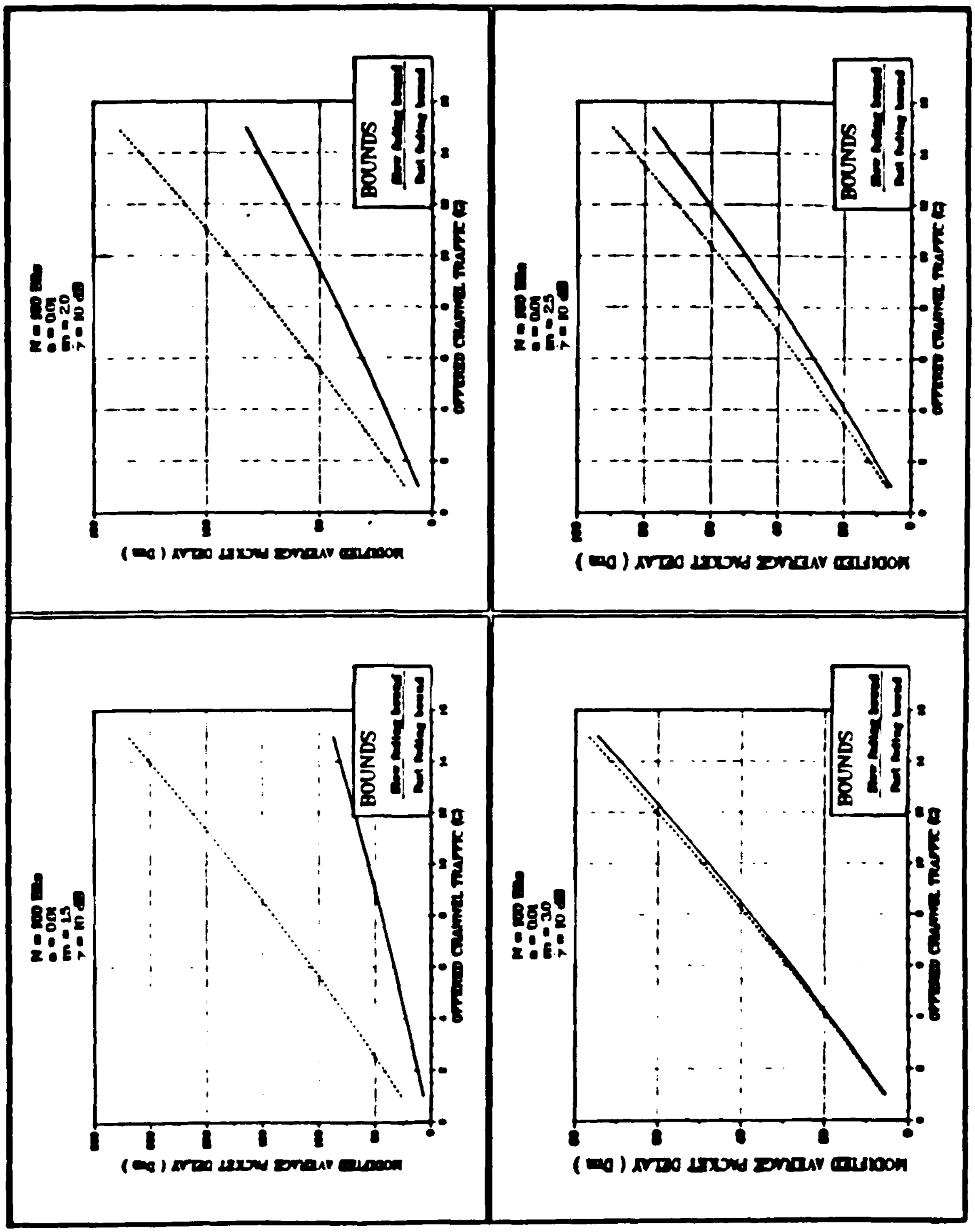

Figure 4 Bounds on the Delay-Traffic characteristics for *NPCSMA* protocol (N=100) and fast *Nakagami* fading for different fading figures.

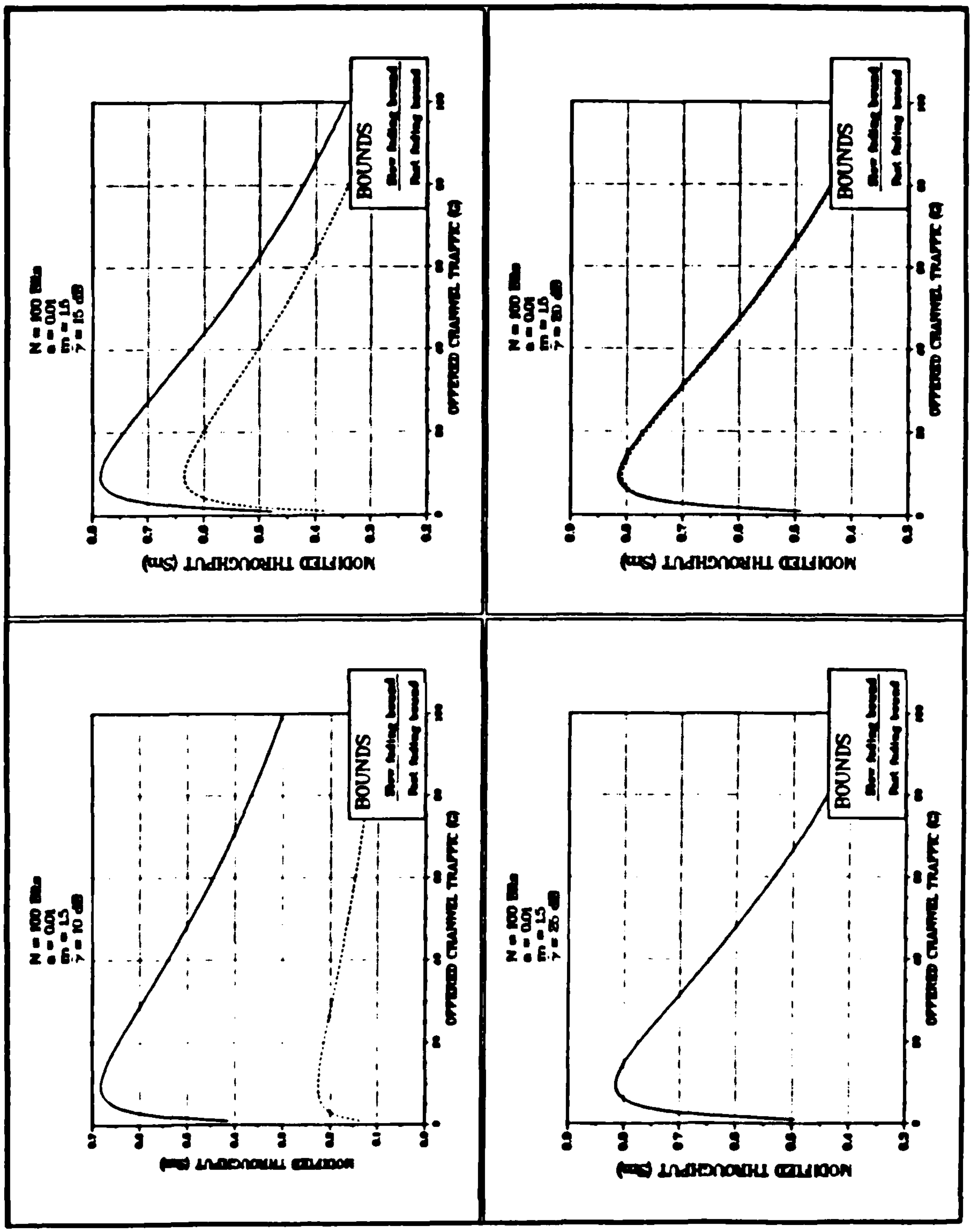

Figure 5 Bounds on the Throughput-Traffic characteristics for *NPCSMA* protocol (N=100) and fast *Nakagami* fading for different *SNRs* .

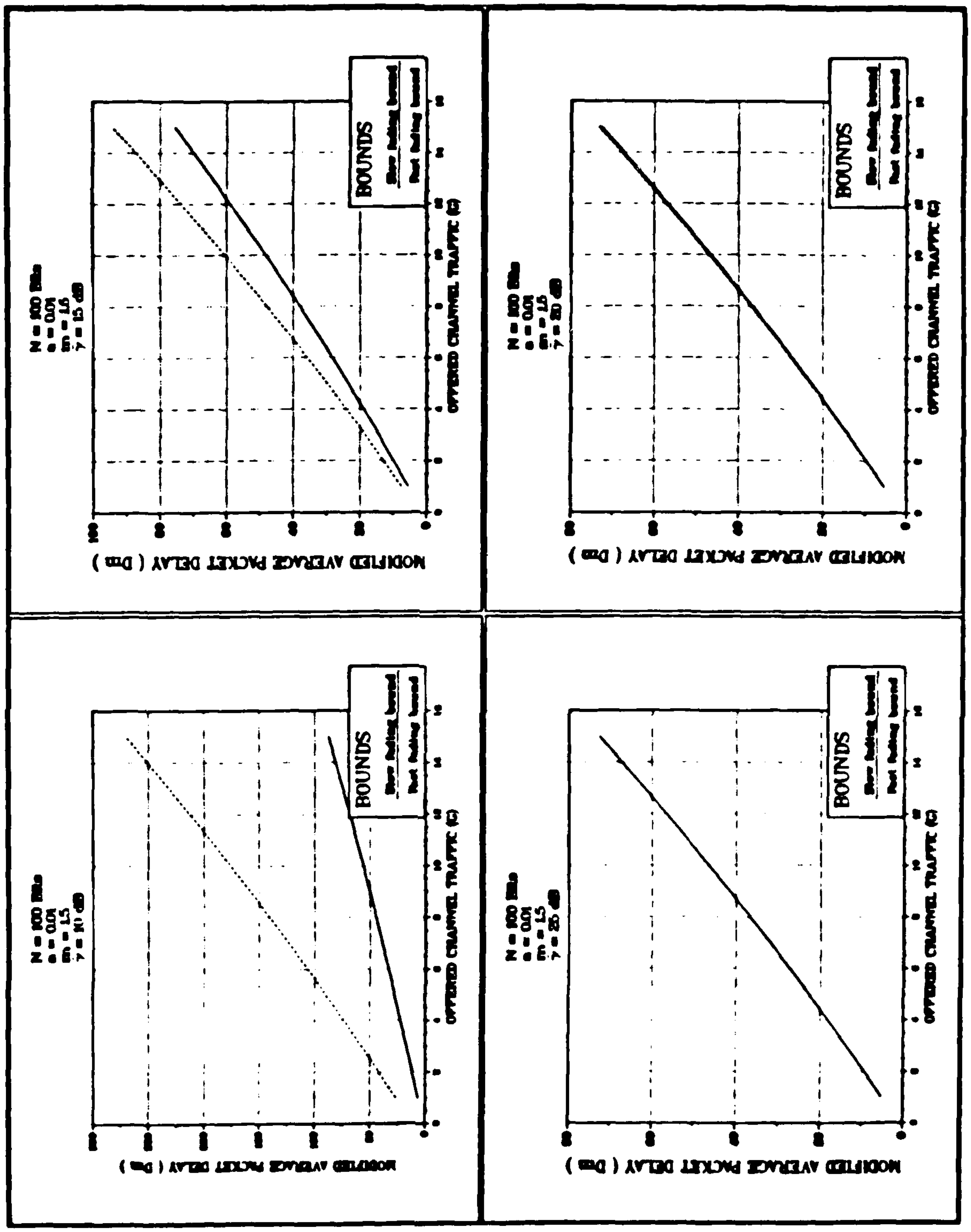

Figure 6 Bounds on the Delay-Traffic characteristics for *NPCSMA* protocol (N=100) and fast *Nakagami* fading for different *SNRs* .

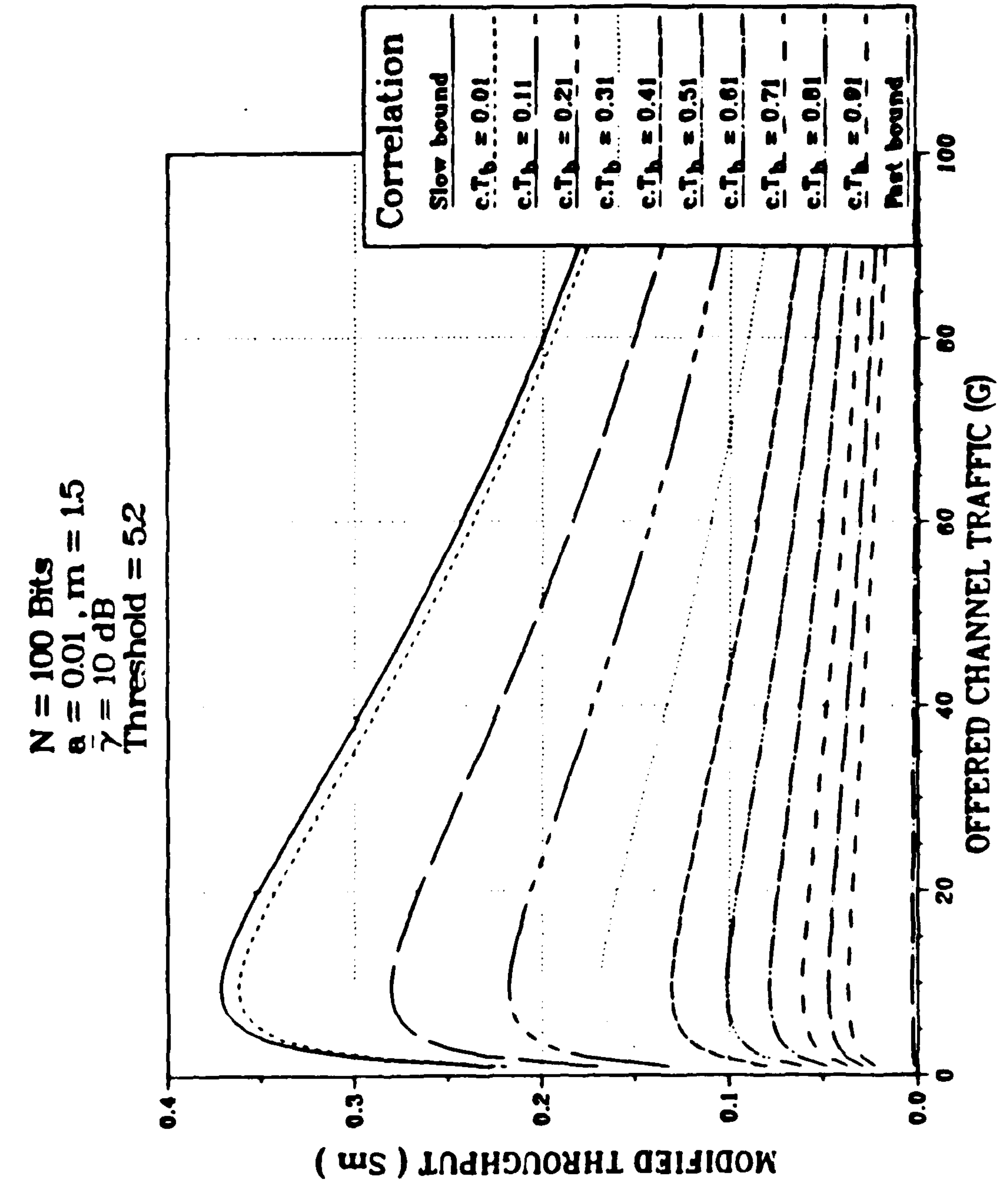

Figure 7 Effect of bit-to-bit correlation on the Throughput-Traffic characteristics for *NPCSMA* protocol (N=100) and intermediate *Nakagami* fading.

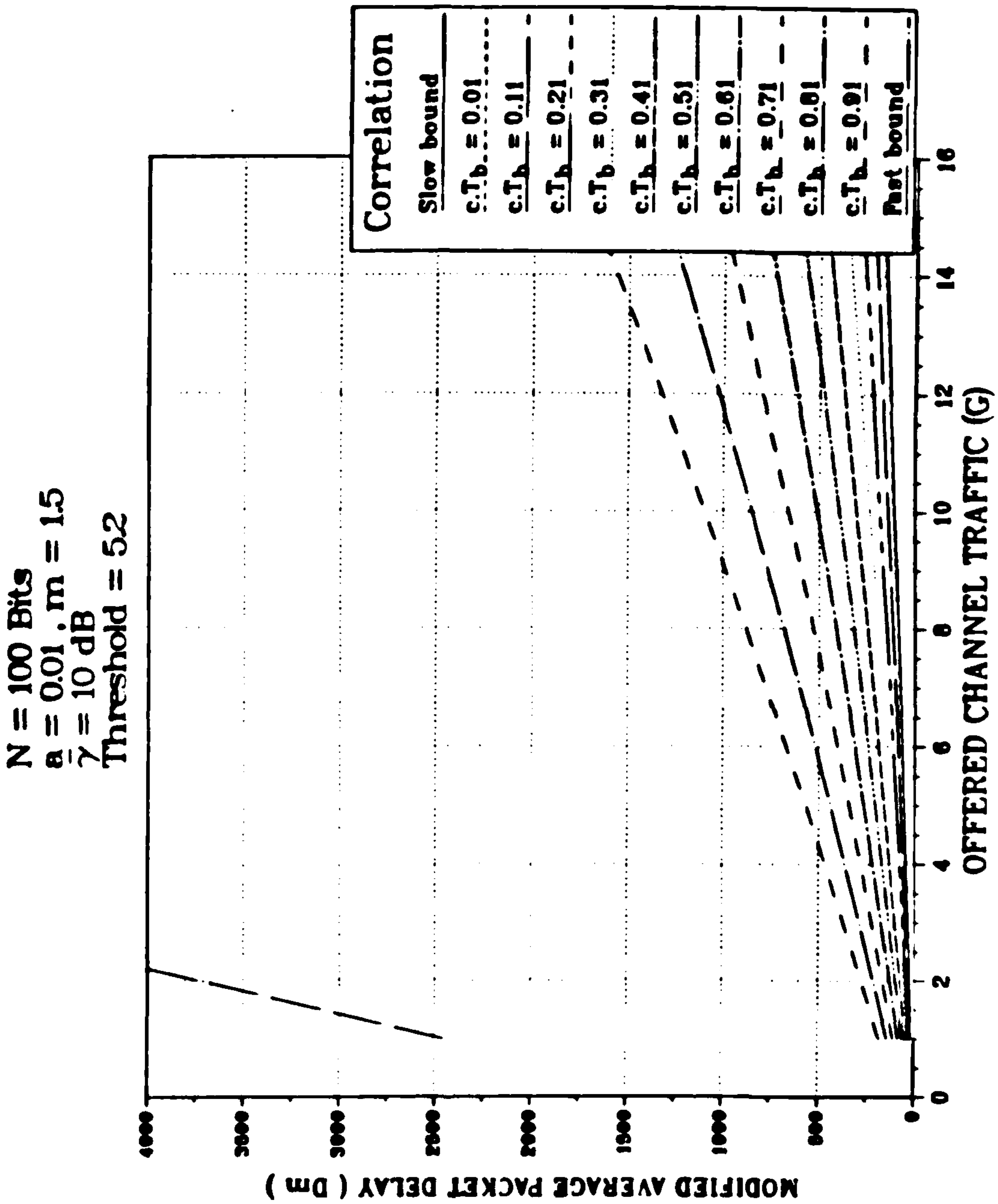

Figure 8 **Effect of bit-to-bit correlation on the Delay-Traffic characteristics for *NPCSMA* protocol (N=100) and intermediate *Nakagami* fading.**

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